**Calculus Syllabus**

1. **Teacher:** Aleksandr Odnakov
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2. **Course description**.
This course is designed to introduce students to the basic ideas and methods of mathematical analysis and their application to mathematical modeling. This course helps lay the foundation for the entire block of quantitative disciplines that are studied at universities, and it also provides some of the analytical tools that are required by advanced courses in economics. This course provides students with experience in the methods and applications of calculus to a wide range of theoretical and practical situations. The course is taught in English
3. **Topic outline.**

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| --- | --- | --- | --- | --- |
| **No.** | **Topic titles** | **TOTAL (hours)** | Contact hours | **Self-study** |
| **Lectures** | **Classes** |
| 1 | Introduction. Functions of one variable. | 8 | 2 | 2 | 4 |
| 2 | Sequences. Limit of a sequence. | 8 | 2 | 2 | 4 |
| 3 | Limit of a function. | 8 | 2 | 2 | 4 |
| 4 | Continuity. | 8 | 2 | 2 | 4 |
| 5 | The derivative. | 8 | 2 | 2 | 2 |
| 6 | Applications of the derivative. | 24 | 6 | 6 | 12 |
| 7 | Infinite series, power series, Taylor series | 8 | 2 | 2 | 4 |
| 8 | Anti-derivatives and the indefinite integral. | 16 | 4 | 4 | 8 |
| 9 | The definite integral. | 16 | 4 | 4 | 8 |
| 10 | Applications of the definite integral. | 24 | 6 | 6 | 12 |
|  | Total: | **128** | **32** | **32** | **64** |

1. **Assessment.**There are two parts of our course:
Part 1: Topics from 1 to 6 will be covered in this part.
Part 2: Topics from 7 to 10 will be covered in this part.
(all dates are approximate)

At the end of each part the students sit a written exam. Each exam is marked out of 100 points. The weighted sum of the exam marks combined with bonus points for homework and in-class quizzes gives the student’s mark out of 100 pints for each of the semesters. To write final exam students need to hand in all home assignments maybe except two of them and attend midterm exam, if student do not attend midterm exam he must hand in all HA in time to access final exam. The weighted sum of the semester marks gives the student’s final mark out of 100 points for the whole course.
Formula for final grade:
2. **Textbooks and problem books.**Main textbooks.
* Stewart J. (editor), Calculus. Early Transcendentials. 6 edition. Thomson Brooks/Cole, 2008. Any later edition is good as well.
* Dowling E.T. Introduction to Mathematical Economics. McGraw-Hill, 1980.

Main problem books.

* Lockshin J., Calculus: theory, examples, exercises. ICEF
1. Course outline.

1. **Introduction**

 The application of mathematics to describing phenomena. The role of mathematics and mathematical modeling in economics. Different forms of representation of functions. Elementary concepts: domain and range of a function, even and odd functions, periodic functions. Graphs of elementary functions. Shifts and distortions of graphs. Implicit functions. Examples of functions in economics: utility function, production function, cost function, demand and supply functions.

2. **Sequences. Limit of a sequence**

Sequences: bounded and unbounded, infinitely small and infinitely large. Limit of a sequence. Limit theorems for sequences: arithmetic operations, sandwich theorem. Monotone sequences. Convergence of a monotone increasing sequence. The number e.

3. **Limit of a function**

The limit of a function at infinity. Asymptotes of a function at infinity. The limit of a function at a point. Limit theorems for functions. Functions that tend to zero, functions that tend to infinity. First and Second Special Limits. Types of indeterminate forms. Finding limits. Left and right limits.

4. **Continuity**

Definition of continuity of a function at point and on an interval. Continuity of elementary functions. Properties of continuous functions. Points of discontinuity. Classification of points of discontinuity. Vertical asymptotes.

5. **The derivative**

Definition of the derivative. Tangent lines and normal lines. Geometric, physical and economic interpretations of the derivative. Right and left derivatives. Differentiability at a point. Differentiability and continuity. Differentiation. Rules of differentiation. Derivatives of elementary functions. Differentiation of inverse functions. Logarithmic differentiation. Differentiation of implicit functions. Existence of a differentiable implicit function. Definition and geometric interpretation of differentials. Approximate calculations using differentials. The second derivative. The economic meaning of the second derivative. Higher-order derivatives and differentials. Properties of differentiable functions: Rolle's theorem, the Mean Value theorem, Cauchy’s theorem, and their geometric interpretation.

6. **Applications of the derivative**

L’Hospital’s rule. Necessary and sufficient conditions for increasing/decreasing functions. Related rates. Concave and convex functions. Different ways of expressing concavity. Economic interpretation of concave and convex functions. Points of inflection. Local extrema. First-order necessary and sufficient conditions for a local extremum. Second-order necessary and sufficient conditions for a local extremum. Maximum and minimum values of a function on an interval. Geometric and economic applications of optimisation. Curve sketching.

7. **Number series, power series, and Taylor expansions**

Necessary condition for convergence of a series. Harmonic series and power series. The ratio test. Comparing series to test for convergence. Alternating series. Sufficient condition for convergence of an alternating series. Absolute convergence. Radius and interval of convergence of a power series. Abel’s theorems. Taylor’s formula. Taylor and Maclaurin series. Taylor and Maclaurin expansions for elementary functions.

8. **Anti-derivatives and the indefinite integral**

Anti-derivatives. The indefinite integral and its properties. Table of indefinite integrals. Basic methods of integration: direct integration, substitution and integration by parts. Integration of rational functions.

9. **The definite integral**

Problems that require the definite integral. Definition of the definite integral using Riemann sums. Sufficient condition for the existence of the definite integral. Approximate calculation of definite integrals using rectangles and trapezoids. Simpson’s rule. Properties of the definite integral. Differentiation of a definite integral with variable upper bound. The fundamental theorem of calculus. Substitution and integration by parts.

10. **Applications of the definite integral**

Applications of the definite integral in geometry, economics and physics. Area of a flat region, volume of a solid of revolution, volume of a solid with known cross-sections. Use of definite integrals to solve separable differential equations.